Model-based Formulae for λ in ORTH

These are the model-based formulae for λ implemented in the ORTH software.

The TEXoutput generated by Mathematica was formatted manually to produce the formulae given below. Note that any possible errors thus introduced will *not* find their way into the computer code. The computer code that implements these formulae was also generated by Mathematica. The code was verified by comparing its results to those obtained by brute-force computation.

Define $m = n(n-1)/2$.

$$(m-1)\lambda_{BB} = \frac{n(n-1)\rho^2 + (2n-1)\rho + 1}{2\rho^2 + 3\rho + 1} - 1$$

$$(m-1)\lambda_{MN} = \frac{\left(\sqrt{\rho} - 1\right) A}{B} - 1$$

$$(m-1)\lambda_{MAD} = \frac{C}{D} - 1$$

$$\begin{aligned}
A &= A_7\rho^{7/2} + A_6\rho^3 + A_5\rho^{5/2} - A_4\rho^2 - 2A_3\rho^{3/2} + 2A_2\rho - 2\mu(1-\mu)\sqrt{\rho} - 2\mu(1-\mu) \\
B &= 2(1-\rho)(1+\rho)\left(\rho(1-\mu)+\mu\right)\left(1-\mu(1-\rho)\right) \\
A_7 &= n^2(1-2\mu)^2 + n\left(-16\mu^2 + 16\mu - 5\right) + 6\left(3\mu^2 - 3\mu + 1\right) \\
A_6 &= -3n^2(1-2\mu)^2 - 46\mu^2 + 46\mu + n\left(48\mu^2 - 48\mu + 11\right) - 10 \\
A_5 &= 3n^2(1-2\mu)^2 + 54\mu^2 - 54\mu + n\left(-52\mu^2 + 52\mu - 15\right) + 16 \\
A_4 &= n^2(1-2\mu)^2 + 42\mu^2 - 42\mu + n\left(-28\mu^2 + 28\mu - 5\right) + 8 \\
A_3 &= -11\mu^2 + 11\mu + n\left(6\mu^2 - 6\mu + 2\right) - 3 \\
A_2 &= (2n-5)\mu^2 + (5-2n)\mu - 1 \\
C &= \rho C_2 n^2 + \rho C_1 n + 2C_0 \\
C_2 &= -\mu(1-\mu)\rho^2 + \left(6\mu^2 - 6\mu + 1\right)\rho + 3\mu(1-\mu) - 1 \\
C_1 &= \mu(1-\mu)\rho^2 + \left(-22\mu^2 + 22\mu - 5\right)\rho - 3\mu(1-\mu) + 1 \\
C_0 &= \left(9\mu^2 - 9\mu + 2\right)\rho^2 - 2\mu(1-\mu)\rho - \mu(1-\mu) \\
D &= 2(1+\rho)(\rho(1-\mu)+\mu)((1-\rho)\mu - 1)
\end{aligned}$$

Reference:

Zink, R. C. & Qaqish, B. F. (2009). Correlated binary regression using orthogonalized residuals. COBRA Preprint Series 51. URL http://biostats.bepress.com/cobra/ps/art51.